Geometric Distributions (Solutions)

1. A fair dice is thrown and its score noted.
   a. If X is the distribution of the scores obtainable, state the distribution of X and any parameters. [1]
   b. Calculate E(X). [2]
   c. Calculate Var(X). [2]

a/ $X \sim U(6)$

b/ $E(X) = \dfrac{6+1}{2} = \dfrac{7}{2}$

c/ $Var(X) = \dfrac{1}{12}(6^2 - 1) = \dfrac{35}{12}$

2. A fair spinner has edges numbered 2, 4, 6 and 8. If Y is the score obtained on any one spin of the spinner:
   a. Find E(Y) [2]
   b. Find Var(Y) [3]

a/ $X \sim U(4)$

$E(X) = \dfrac{4+1}{2} = \dfrac{5}{2}$

$\Rightarrow E(Y) = E(2X) = 2E(X) = 5$

b/ $Var(X) = \dfrac{1}{12}(4^2 - 1) = \dfrac{15}{12} = \dfrac{5}{4}$

$Var(Y) = Var(2X) = 2^2 Var(X) = 4\left(\dfrac{5}{4}\right) = 5$

3. A fair dice with faces numbered 3, 4, 5, 6, 7 and 8 is thrown. Find the expectation and the variance of the score obtained on throwing the dice. [5]

$X \sim U(6) \Rightarrow E(X) = \dfrac{6+1}{2} = \dfrac{7}{2}$, $Var(X) = \dfrac{1}{12}(6^2 - 1)$

$= \dfrac{35}{12}$

$\Rightarrow E(X+2) = E(X) + 2 = \dfrac{11}{2}$ $Var(X+2) = Var(X) = \dfrac{35}{12}$

4. A mini-roulette is spun. The possible outcomes are 5, 8, 11, 14 and 17. Find the expectation and variance of the average score obtained over a large number of spins. [6]

$X \sim U(5) \implies E(X) = \frac{5+1}{2} = 3, \quad Var(x) = \frac{1}{12}(5^2 - 1) = 2$

$E(3X + 2) = 3E(x) + 2 = 11, \quad Var(3X + 2) = 3^2 Var(x) = 18$

5. A variable has a uniform distribution with n possible outcomes. Given that the variance of the distribution is 10, find the value of n and the expectation of the distribution. [4]

$Var(x) = 10 \implies \frac{1}{12}(n^2 - 1) = 10 \implies n^2 - 1 = 120$

$\implies n^2 = 121 \implies n = 11$

$\therefore E(x) = \frac{11 + 1}{2} = 6$